Steady-State Interactions in Plasma Accelerators

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Theme

AN analytical study of the steady-state interactions between a gas and a force field is presented. The several different types of interactions that occur for different magnitudes of the body force and heat addition are described. The effect of maintaining an initial pressure difference across the force field is also discussed. Although the analysis is applicable for any type of body force and heat input, reference is made to plasma accelerators where the Lorentz force and Ohmic heating are the appropriate quantities.

Contents

A stationary, nondiffusive, one-dimensional force field is assumed to have been created in an infinitely long channel initially containing a perfect gas at rest. A single-fluid model is used to investigate the effect of the force field on the gas. The difficulties of such an investigation are reduced considerably by only studying the steady state, which is defined as the state in which the shock and expansion waves have constant strengths and steady-state conditions exist inside the force field. With these approximations the equations that govern the flow of the gas through the force field are derived and are solved along with the equations for shock and expansion waves to determine the resulting interaction. A related problem, that of the interaction of a shock tube flow with an electromagnetic field, has been treated analytically by Johnson,[1] who used the method of characteristics and by Rosciszewski and Gallaher,[2] who used the Lax-Wendroff finite difference method. Experimental results have been reported by Zauderer and Tate.[3]

Two different types of force field are considered. For the first type the force field is independent of the gas velocity through it and for the other it is altered by the gas velocity. Each of these types is related to a particular class of plasma accelerators.

For the case of a force field independent of the gas velocity the possibility of placing a diaphragm at the forward edge of the force field that is burst at the same instant the force field is created is also considered. The parameter formed by this initial pressure difference is represented by P and is equal to the initial pressure ahead of the diaphragm divided by the initial pressure behind it (on the force field side). The gas temperature and the type of gas are assumed to initially be the same on each side of the diaphragm.

The seven possible interactions between the gas and the force field in the steady state are indicated schematically in Fig. 1. The force and initial pressure difference accelerate the gas to the right in the figure. The regions in which the flow variables have constant values for each interaction are indicated by the circled numbers, and the numerical subscripts indicate the region referred to. The velocities indicated by the arrows are the velocities of the shocks, contact surfaces and edges of the expansion waves when viewed from a laboratory frame.

Two shocks are formed for the type II, IV, and VII interactions, and the second shock remains inside the force field for the type II and VII interactions. Expansion waves between the shock and the force field are created for the type III, V, and VI interactions. The left edge of this expansion remains stationary at the edge of the force field (flow is sonic at this point) for the type VI interaction. The right edge of the expansion wave behind the force field is stationary for the type II, III, and IV interactions.

A force field independent of the gas velocity is approximated in plasma accelerators in which the force field is created by the interaction of a current with a self-induced magnetic field. In

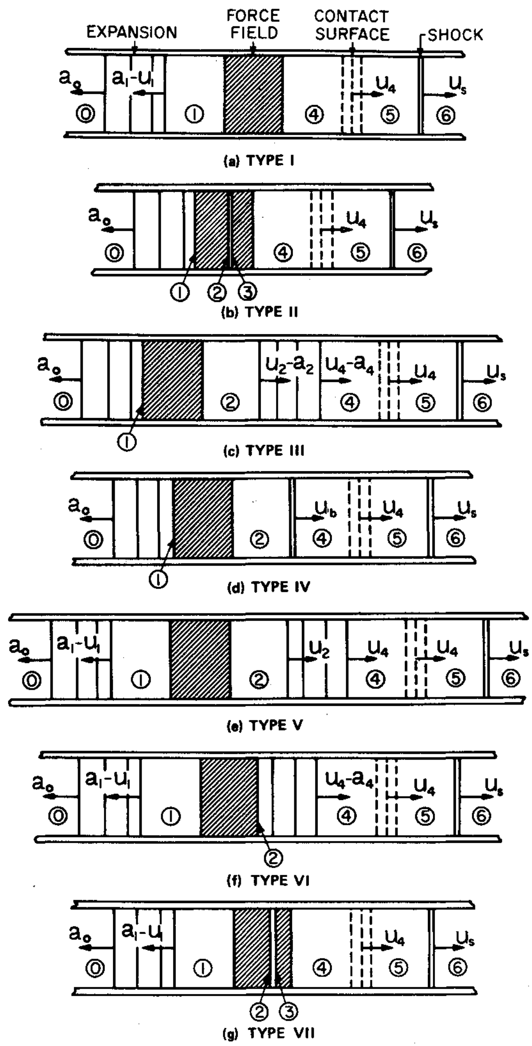

Fig. 1 Steady-state interactions.

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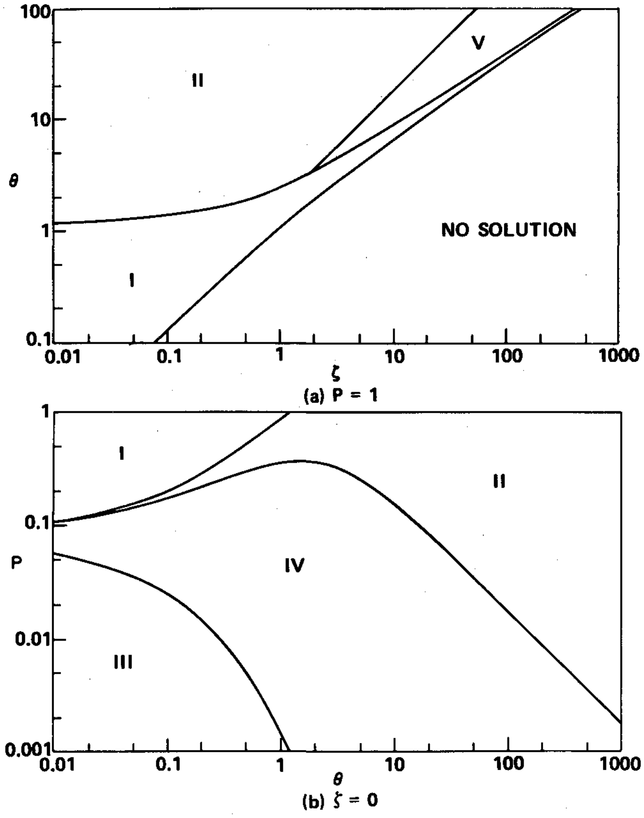

Fig. 2 Regions for which each steady-state interaction exists for a force field independent of the gas velocity ($\gamma = 5/3$).

order that this analysis be applicable to this class of accelerators, it must be assumed that the Lorentz force and Ohmic heating are known quantities that can be treated as parameters. The electrical conductivity must also remain low enough that the magnetic lines of force do not become frozen into the gas. For the above type of force field all of the interactions except the type VII interaction may exist. The boundaries of the regions for which each type of interaction occurs for specified values of one of the three parameters (θ, ζ, P) are shown in Fig. 2. The type VI interaction occurs for low P and moderate to high ζ values and is not seen in Fig. 2 (see Fig. 3 of the full paper). The parameters θ and ζ are defined by

$$\theta = 2Fx/\gamma p_0 \qquad \zeta = Hx/\gamma p_0 a_0$$

where F is the body force, H is the heat addition, p_0 and a_0 are the initial pressure and sound speed on the high pressure side of the diaphragm, x is the width of the force field and γ is the specific heat ratio.

For no initial pressure difference across the force field, we can see from Fig. 2a that the type II interaction occurs for a large range of conditions that are achieved in experimental devices (large values of the body force and relatively small values of the heat input). Thus there should be a shock inside the force field in these devices in addition to the shock ahead of the accelerated gas. The stationary shock inside the force field is, in general, much stronger than the propagating shock. In fact, if the strong shock approximation is valid for both shocks and if $\theta \gg 1$, the temperatures in regions 4 and 5 are

$$T_4 = T_0\gamma[(\gamma+1)/2]^{(\gamma+3)/2(\gamma-1)}(\theta/2)^{3/2}$$

$$T_5 = T_0[\gamma(\gamma-1)/2(\gamma+1)]\theta$$

Hence the shock inside the force field causes the gas temperature between the force field and the contact surface to be much larger than the temperature between the contact surface and the propagating shock. This may offer an explanation as to why some experimenters mistook the contact surface for the propagating shock. Approximate analytic expressions are easily derived for the shock velocity and steady-state gas velocity for the type II interaction with the above approximations, and the results are, respectively,

$$u_s = a_0[(\gamma+1)\theta/4]^{\frac{1}{2}} \qquad (1a)$$

$$u_{ss} = a_0[\theta/(\gamma+1)]^{\frac{1}{2}} \qquad (1b)$$

The steady-state gas velocity is the velocity that exists ahead of the force field after the shock and expansion waves have propagated downstream.

Maintaining a large initial pressure differential across the diaphragm in devices for which the heat addition is negligible causes either the type III or type IV interaction to occur as shown in Fig. 2b. For the assumptions used in deriving Eqs. (1) and for $P \ll 1$, the shock and steady-state gas velocities for both the type III and type IV interactions become

$$u_s = a_0[(\gamma+1)/2]^{2\gamma/(\gamma-1)}\theta/2 \qquad (2a)$$

$$u_{ss} = a_0[(\gamma+1)/2]^{(\gamma+1)/(\gamma-1)}\theta/2 \qquad (2b)$$

Note that the above velocities do not depend on the actual value of P. By comparing Eqs. (1) and (2), we see that a large initial pressure difference across the diaphragm causes these velocities to change from a square root to a linear dependence on the magnitude of the body force parameter.

The possibility of placing a diaphragm at the forward edge of the force field is not considered for the case of a force field altered by the gas velocity through it. This type of force field occurs in plasma accelerators in which the force field is created by the interaction of externally applied electric and magnetic fields. In the present analysis the externally applied fields are assumed to be constant, and any self-induced magnetic fields are assumed to be negligible compared to the externally created field. The type I, II, and VII interactions may exist in this case, and it can be shown that the type VII interaction exists for most conditions of interest. The shock and steady-state gas velocities for the type VII interaction are

$$u_s = [(\gamma^2-1)/4\gamma]E_0/B_0 + (\{[(\gamma^2-1)/4\gamma]E_0/B_0\}^2 + a_0{}^2)^{\frac{1}{2}}$$

$$u_{ss} = [(\gamma-1)/\gamma]E_0/B_0$$

where E_0 and B_0 are the externally applied electric and magnetic fields, respectively.

References

[1] Johnson, M. R., "Shock-Induced Flow Interaction with Transverse Magnetic Fields," *The Physics of Fluids*, Vol. 10, No. 3, March 1967, pp. 539–546.

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